

# Scaling Law for the Magnetic Field of the Planets Based on a Thermodynamic Model

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## ABSTRACT

A thermodynamic model for the generation of magnetic fields in the planets is proposed, considering crossed effects between gravitational and electric forces. The magnetic field of the Earth is estimated and found to be in agreement with the actual field. The ratio between the field of several planets and that of the Earth is calculated in the model and compared with the same ratio for the measured fields. These comparisons are found to be qualitatively consistent. Once the value of the magnetic field is calculated, the model is used to obtain the tilt of the magnetic dipole with respect to the rotation axis. This model can explain why Uranus and Neptune magnetic fields have higher quadrupole moment than the other magnetic fields of the Solar System and why Saturn, that has a highly axysymmetric field, has lower quadrupolar component. The model also explains the double peak of the magnetic field observed by Voyager 2 while recording the field of Neptune. The Earth paleomagnetic data are analysed and found to be consistent with the model, that predicts higher quadrupole components for the more tilted dipoles. A field is predicted for all the planets and satellites of the Solar System with enough mass. Objections are made to the theories that predict that this effect could not generate a field agreeing with the measured one.

*Subject headings:* MHD, planets and satellites: general, magnetic fields

## 1. Introduction

One of the open fields in modern physics is to find the equations that rule the origin and evolution of the magnetic field of the planets and, specifically, that of the Earth. Some theories based on the dynamo model have obtained a correct value for the magnetic field for the Earth (Glatzmaier & Roberts (2000); Busse (2000)), and even some of the proposed models describe a magnetic pole inversion as observed in the Earth (Glatzmaier & Roberts (1995b,a); Plunian et al. (1998)). The dynamo model has also been applied to describe the magnetic field of the Sun (Glatzmaier (1985)). Even though these theories seem to fit correctly to the field of our planet, there is a question still unsolved. Why all the objects of the Universe seem to be closely related with magnetic fields?. Six of the nine planets of our solar system, main sequence stars, neutron stars, galaxies, etc..., all seem to have high magnetic fields.

If we look in further detail to the Solar System, the presence of significative magnetic fields seems to be the law rather than the exception (Russell (1993a,b)). But this ubiquity in the presence of magnetic fields does not seem to remain if we look at the morphologies of such fields. Mercury and Jupiter show Earth-like structure fields (Ness et al. (1979)). Uranus (Connerey & Acua (1987)) and Neptune very inclined fields and quadrupolar moments. In the case of Neptune, the Voyager 2 probe recorded a double peak in the mesured magnetic field (Connerey & Acua (1991)). Saturn, in contrast, has a highly axisymmetric magnetic dipolar field (Acua et al. (1983)).

Although the geodynamo model explains so well the Earth magnetic field, it must make nontrivial assumptions to incorporate subtle details in the magnetic fields of other planets. For instance, Uranus and Neptune’s magnetic fields are more quadrupolar than the one predicted by the outer core convective dynamo. In the same manner paleomagnetic data in the Earth show an increment of the relative intensity of the quadrupolar components of the magnetic fields during the magnetic dipole reversals. Another problem is that Saturn’s strongly axisymmetric field contradicts Cowling’s theorem that no axisymmetric homogeneous magnetic field can be self-sustained. There have been solutions for each of these problems. The quadrupolar problem of Uranus and Neptune has been solved by a change of the convection zone. In the modified model, the convection is produced in a zone between 0.75 and 0.8 times the radius of the planet (Aurnou (2004); Stanley & Bloxham (2004)). The model proposed uses a fluid electrically-conducting inner core, instead of the solid electrically-conducting inner core of the standard geodynamo models. The high axysimmetry of Saturn has been solved in models like Stevenson (1982). All these solutions make the theory not easily applicable to all different cases in the Solar System with one simple model. Although dynamo theory is the best accepted way to fit to the experimental data, the models used to describe

these facts are different for every object described. This paper tries to solve this problem with the use of a model that tries to explain all these features without the need specificities.

In this paper we propose a model of magnetic field generation based on a thermodynamic point of view which uses the relation between thermodynamic forces and fluxes as the starting point. In the model, a redistribution of the charge is obtained from the gravitational energy of the planet through the action of pressure. From this charge distribution and the rotation of the planet, the magnetic field is generated. Several advantages of this model are: 1) Its ability to spontaneously break the spherical symmetry of the system to yield a field with axial symmetry; 2) the possibility to obtain a tilt of the dipole moment relative to the axis of rotation; 3) a possible explanation of why Uranus and Neptune, that have large tilts, have higher quadrupole moment than the other planets; 4) an explanation of why Saturn, with a high axisymmetric field, has a low quadrupole moment.

The possibility of reversing the dipole is open, and predicts that the field during a transition may become more quadrupolar than the stable normal or reverse state (as observed in the past). The detailed dynamics of the transition is out of the scope of this paper that only pretends to present the main lines of the model.

The inclusion of an analysis of the multipolarity of the fields obtained is also out of the scope of this article, but this feature is implicitly considered in the model because of its geometry.

This paper doesn't attempt to replace the geodynamo model that fits so well in the case of the Earth (Glatzmaier & Roberts (2000, 1996)), but tries to propose a general mechanism for the different planets to generate a magnetic field. Instead of it, the energy needed for the generation of the field comes from the more universal gravitational field, present all over the Universe. To describe in an unified way the behaviors of such different planets, it is logical to ask for a model relatively independent of the microscopic details, this is the reason why we have turned here to a global thermodynamic analysis, at a macroscopic level, rather than directly going to the mechanistic details, which may be different in different planets.

The model could be used in posterior papers, as a generator of magnetic fields in more general objects of the Universe like the main sequence stars or the neutron stars.

## 2. Model

We start our discussion by calculating the conditions of pressure in the core. The pressure inside the planet is given in the first approximation by the hydrostatic equilibrium

between gravitation, that pushes inwards, and pressure, that pushes outwards. The resulting value of the pressure distribution is

$$p(r) = \frac{9GM^2}{8\pi} \left( \frac{1}{R^4} - \frac{r^2}{R^6} \right), \quad (1)$$

where  $M$  and  $R$  are the total mass and external radius of the planet and  $r$  is the radius at the position being considered. In this relation it is assumed that the density is constant and equal to the average density of the planet. This prevents the gravitational collapse of the planet. We must take into account that we don't pretend to have an exact value for this parameter, but only an idea of its magnitude and we don't need a more accurate value of it.

The internal pressure of the Earth makes iron in the inner part of the core to be more compact packed than iron in the outer part. This increase in atomic packing makes the Fermi energy in the inner part of the core to be higher than the one in the outer part. This difference in the Fermi energy pushes the electrons in the conduction band outside the center of the Earth. If the charge of the materials were zero (conducting iron in the inner core and electrons pushed outside), this tendency would continue until the complete separation of both particles, but the particles are charged and we face a problem of stratification with electrical charge. In this scenario, there is no need of ionization of the core's material, the only assumption made is its conductivity, and the fact that the conduction electrons have complete mobility in all the conductor. The property of conductivity is common to all the planet because in the core all the planets have high enough pressures to cause the constituent material to act as a metal even though the material is hydrogen like in Jupiter and Saturn.

Thermodynamically we could see the model as a gradient of chemical potential due to a pressure gradient. Electrons in a material are subject to the force exert by the gradient of the electrochemical potential. This gradient has the form

$$\nabla \mu_{el} = \mathbf{E} + \nabla \mu, \quad (2)$$

where  $\mathbf{E}$  is the external electric field and  $\mu_{el}$  and  $\mu$  are respectively the electrochemical and chemical potentials.

A gradient of chemical potential could be caused by several reasons. We could obtain a gradient by changing the concentration, the pressure and the temperature. Thermodynamically, the cause of an electric current is the electrochemical gradient. If there is no current present in a material, the gradient of electrochemical potential must be zero. If we take into account all these generators in the last relation and impose the equilibrium condition, we obtain

$$\frac{\partial \mu}{\partial p} \nabla p + \frac{\partial \mu}{\partial T} \nabla T + \frac{\partial \mu}{\partial c} \nabla c = 0, \quad (3)$$

where  $p$  is the pressure,  $T$  temperature and  $c$  concentration.

In the radial direction in the Earth core, the conservation of charge grants that the net current over sufficient long periods of time must be zero. This condition leave only the possibility that the radial current is zero or oscillate around this value. We start our discussion by taking the firs possibility. The main cause of

In this simple model we assume two concentric spherical shells filled with some substance with electric permittivity  $\epsilon$ , electric conductivity  $\sigma$ , and thermal conductivity  $\kappa$ . These shells corresponds to the inner core boundary (ICB) and the core mantle boundary (CMB).

The energy balance equation of the material filling the system is, if we consider electrical effects,

$$\rho \dot{u} = -\nabla \cdot \mathbf{q} + \mathbf{i} \cdot \mathbf{E}, \quad (4)$$

where  $\rho$  is the mass density,  $u$  internal energy per unit mass,  $\mathbf{i}$  electrical current density,  $\mathbf{q}$  the energy flux and  $\mathbf{E}$  the electric field. The conservation of mass of component  $k$  is expressed by

$$\rho \dot{c}_k = -\nabla \cdot \mathbf{j}_k, \quad (5)$$

with  $c_k = \rho_k / \rho$  the mass fraction and  $\mathbf{j}_k$  the material current density.

The classical entropy of the system is defined with the usual Gibbs equation

$$ds = \frac{1}{T} du - \frac{p}{T} dv - \sum_{k=1}^n \frac{\mu_k}{T} dc_k. \quad (6)$$

By differentiating this equation with respect to time, and substituting (4) and (5) in the subsequent equation we obtain

$$\rho \dot{s} = \frac{1}{T} \left( -\nabla \cdot \mathbf{q} + \mathbf{i} \cdot \mathbf{E} \right) - \frac{p\rho}{T} \dot{v} - \sum_{k=1}^n \frac{\mu_k}{T} \nabla \cdot \mathbf{j}_k. \quad (7)$$

Arranging terms, we obtain four fluxes and four generalized forces appearing in the entropy production, namely, the last four terms in the right hand side of

$$\begin{aligned} \rho \dot{s} = & -\nabla \cdot \left( \frac{\mathbf{q} - \sum_{k=1}^n \mu_k \mathbf{j}_k}{T} \right) + \left( \mathbf{q} - \sum_{k=1}^n \mu_k \mathbf{j}_k \right) \cdot \nabla \left( \frac{1}{T} \right) + \\ & + \mathbf{i} \cdot \frac{\mathbf{E}}{T} - \frac{p\rho}{T} \dot{v} - \sum_{k=1}^n \frac{\nabla \mu_k}{T} \cdot \mathbf{j}_k \end{aligned} \quad (8)$$

Flux	Generalized force
$\dot{v}$	$\rho p/T$
$\mathbf{j}_k$	$\nabla \mu_k/T$
$\mathbf{q} - \sum_{k=1}^n \mu_k \mathbf{j}_k$	$\nabla(1/T)$
$\mathbf{i}$	$\mathbf{E}/T$

Table 1: table showing the fluxes and the corresponding generalized forces appearing in the entropy production in Eq (8)

As it is known, a generalized force can produce not only its associated flux, but also other fluxes, like in thermoelectricity or thermodiffusion (de Groot & Mazur (1962); Jou et al. (2001)). In our example, we analyse the coupling between electric field and the gradient of chemical potential. The constitutive equations of the corresponding fluxes are

$$\mathbf{i} = L_{ee} \frac{\mathbf{E}}{T} + L_{eg} \frac{\nabla \mu_k}{T}, \quad (9)$$

$$\mathbf{j}_k = L_{ge} \frac{\mathbf{E}}{T} + L_{gg} \frac{\nabla \mu_k}{T}, \quad (10)$$

where  $L_{ee}$  is related to the electrical conductivity of the material ( $L_{ee} = \sigma T$ ),  $L_{gg}$  is related to the diffusion coefficient of component  $k$  and  $L_{eg} = L_{ge}$  is the crossed coefficient that relates the electric and gravitational effects. We use the  $g$  subscripts because, below, this value will be related with the gravitational energy.

The gradient of the chemical potential is related with the pressure gradient as

$$\nabla \mu_k = v_k \nabla p, \quad (11)$$

where  $v_k$  is the specific molar volume of species  $k$ . We consider that the dependence of  $\mu_k$  on the temperature is negligible as compared with that on the pressure. Using this relation and an approximate value of the gradient of  $p$  obtained from (1), we get the following relation between the fluxes,

$$\mathbf{i} = L_{ee} \frac{\vec{\mathbf{E}}}{T} + L_{eg} \frac{v_k}{T} \left( \frac{9GM^2}{8\pi} \frac{1}{R^5} \right) \hat{\mathbf{r}}, \quad (12)$$

$$\mathbf{j}_k = L_{ge} \frac{\vec{\mathbf{E}}}{T} + L_{gg} \frac{v_k}{T} \left( \frac{9GM^2}{8\pi} \frac{1}{R^5} \right) \hat{\mathbf{r}}, \quad (13)$$

with  $\hat{\mathbf{r}}$  the unit vector in the radial direction.

If we consider that the planet interior is in equilibrium, the radial electrical and material currents must vanish, in such a way that there must be an electrical field that opposes to the gravitational effects on the charged fluid

$$L_{ee}\frac{\vec{\mathbf{E}}}{T} + L_{eg}\frac{v_k}{T}\left(\frac{9GM^2}{8\pi}\frac{1}{R^5}\right)\hat{\mathbf{r}} = 0, \quad (14)$$

which yields

$$\vec{\mathbf{E}} = -\frac{L_{eg}}{L_{ee}}\nabla\mu = -\frac{L_{eg}}{L_{ee}}v_k\left(\frac{9GM^2}{8\pi}\frac{1}{R^5}\right)\hat{\mathbf{r}}. \quad (15)$$

Now it is time to examine the effect of this electrical field on the electrons of the core. Figure 2 shows the model we use. We can see it like a condenser composed of two spherical plates of radius  $r_a = \alpha R$  (inner sphere) and  $r_b = \beta R$  (outer sphere), where  $R$  is the planet radius and  $\alpha$  and  $\beta$  are numerical coefficients which take a value between 0 and 1. We suppose that the planet stores electrical charge until the field created by this charge cancels the one created by gravitational effects.

To calculate the charge we need the value of the capacity of the condenser (Wangsness (1992)). This value is

$$\frac{1}{C} = \frac{1}{4\pi\epsilon}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{1}{4\pi\epsilon R}\left(\frac{\beta - \alpha}{\beta\alpha}\right) \quad (16)$$

The value of the electrical field created by this condenser is related with the charge stored in it. If we suppose that the value of the electric field is approximately constant inside the plates and it is given by (15) we obtain for the charge

$$\begin{aligned} Q &= C\Delta V = C\mathbf{E}(r_b - r_a) = \\ &= 4\pi\epsilon R\left(\frac{\beta\alpha}{\beta - \alpha}\right)\frac{L_{eg}}{L_{ee}}(\nabla\mu)R(\beta - \alpha) = \\ &= 4\pi\beta\alpha\frac{L_{eg}\epsilon}{L_{ee}}(\nabla\mu)R^2 = C_g C_m (\nabla\mu)R^2 \end{aligned} \quad (17)$$

where  $C_g = 4\pi\beta\alpha$  is a value that only depends on the geometry of the system and is independent of the values of the material coefficients, whereas, in contrast,  $C_m = L_{eg}\epsilon/L_{ee}$  is related only with the values of the conductivities of the material contained between the plates.

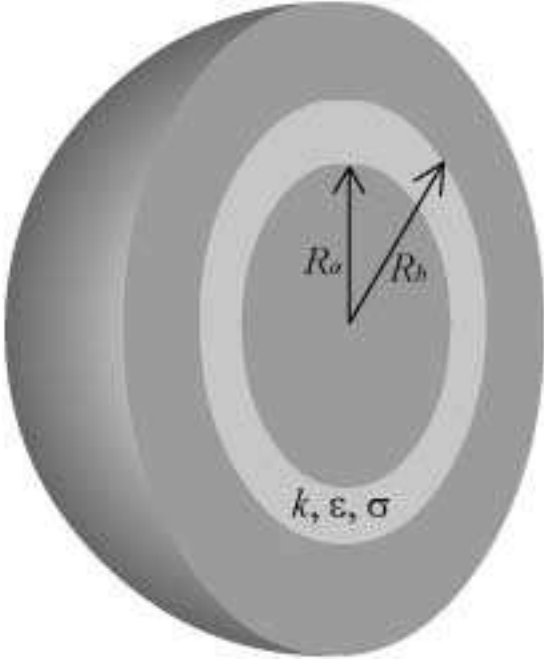


Fig. 1.— Sketch of the interior of a planet with the symbols being used in the text

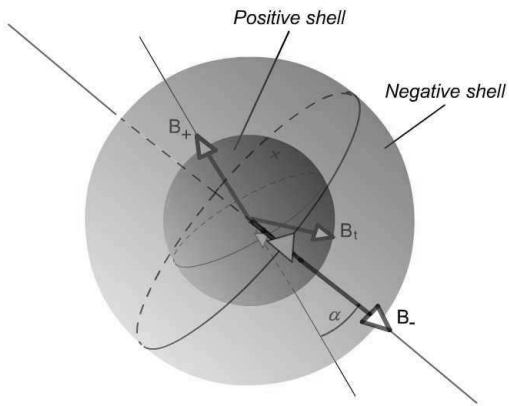


Fig. 2.— Image of the magnetic field created by each one of the different shells and the total magnetic field.



If we distribute the charge  $Q$  uniformly all over the plates we find the values of the respective charge density  $\sigma_a$  and  $\sigma_b$  of the inner and outer spheres

$$\sigma_a = \frac{C_g C_m \nabla \mu R^2}{4\pi \alpha^2 R^2} = \frac{C_g C_m \nabla \mu}{4\pi \alpha^2}, \quad (18)$$

$$\sigma_b = -\frac{C_g C_m \nabla \mu R^2}{4\pi \beta^2 R^2} = -\frac{C_g C_m \nabla \mu}{4\pi \beta^2}. \quad (19)$$

These densities will rotate with the planet with a frequency  $\nu = (1/\tau)$  where  $\tau$  is the corresponding rotation period. These charges, rotating with this angular velocity will generate a dipolar magnetic field. In the case of the internal plate this magnetic dipole moment is

$$\begin{aligned} d_a &= \int \mathbf{S} d\mathbf{i}_a = \int \mathbf{S} \sigma_a \nu d\mathbf{A}_a = \\ &= \int_0^\pi \pi (R\alpha \sin \theta)^2 \sigma_a \nu (2\pi \alpha R \sin \theta) R d\theta = \\ &= \frac{2\pi \alpha^2 C_g C_m R^4}{3} \frac{\nabla \mu}{\tau} \mu_k \end{aligned} \quad (20)$$

while in the case of the outer plate we obtain

$$d_b = -\frac{2\pi \beta^2 C_g C_m R^4}{3} \frac{\nabla \mu}{\tau}. \quad (21)$$

The total magnetic moment will be the sum of (20) and (21) values. Thus the final dipole moment of the planet is

$$d_t = d_b + d_a = -\frac{2\pi(\beta^2 - \alpha^2)C_g C_m \nabla \mu R^4}{3} \frac{1}{\tau}. \quad (22)$$

The only remaining thing is to return to the value of the gradient of the chemical potential in terms of the mass and radius of the planet, given by (15), i.e.

$$\nabla \mu_k = v_k \left( \frac{9GM^2}{8\pi} \frac{1}{R^5} \right). \quad (23)$$

If we substitute this value in (22) we obtain finally

$$\mathbf{d}_t = C'_g C'_m G \frac{M^2}{R\tau}, \quad (24)$$

where we have redefined the geometrical and material factors as  $C'_g = -3\pi(\beta^2 - \alpha^2)\beta\alpha$ , and  $C'_m = \frac{L_{eg}v_k\epsilon}{L_{ee}}$ .

Equation (24) gives the total magnetic field generated by an object of mass  $M$ , radius  $R$  and that rotates with a period  $\tau$ .

The resulting magnetic field is a linear combination of two dipoles: the one produced by the positive charge in the inner shell and the one produced by the negative charge in the outer shell. Both shells rotate in the same direction and, as a consequence, the magnetic fields generated by them are in opposite directions as they have opposed electric charges. The resulting dipole is in the direction of the bigger one, in this case in the direction of that generated by the negative charges because their linear velocity is higher than that of the positive charges.

At this point we must solve the possibility of the model to predict a change in the orientation of the generated magnetic field in order to fit to the experimental data recorded in basaltic rocks.

For this purpose we have to come back to the thermodynamic relationship and observe the behavior of the fluxes in the toroidal component. In this case we have to take on consideration that forces due to the magnetic field appear in the formulas. Expressed in tensorial form, constitutive equations are

$$\mathbf{E} = \mathbf{i}/\sigma + \alpha \nabla T + R \mathbf{H} \times \mathbf{i} + N \mathbf{H} \times \nabla T, \quad (25)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic field,  $\mathbf{i}$  is the electric current density,  $T$  is temperature and  $\alpha, \sigma, R, N$  are respectively the thermal and electrical conductivity and Hall and Nernst conductivity. In this section we are only interested in the behavior of this relation in the toroidal component. We could make some simplifications. Since the system has radial symmetry, the gradient of temperature and the electric field in this direction are 0, ( $E_\theta = 0, (\nabla T)_\theta = 0$ ). In the previous section we considered that the electric current in the radial direction is 0, ( $j_r = 0$ ). The magnetic field is in the poloidal component, ( $\mathbf{H}_r = H_\phi$ ). Making these assumptions in the previous relation we obtain

$$0 = j_\theta/\sigma + N H_\phi \nabla T, \quad (26)$$

this relation tells us that in the presence of the magnetic field coming from the radial separation of charges in the core, a current appears in the azimuthal direction due to the gradient of temperatures. This current is in the opposite direction to that of the rotation of the planet. As a consequence, the outer shell "frena" respect to the motion of the planet and this makes the global magnetic field change in magnitude. If this current is high enough, the orientation of the magnetic field could change.

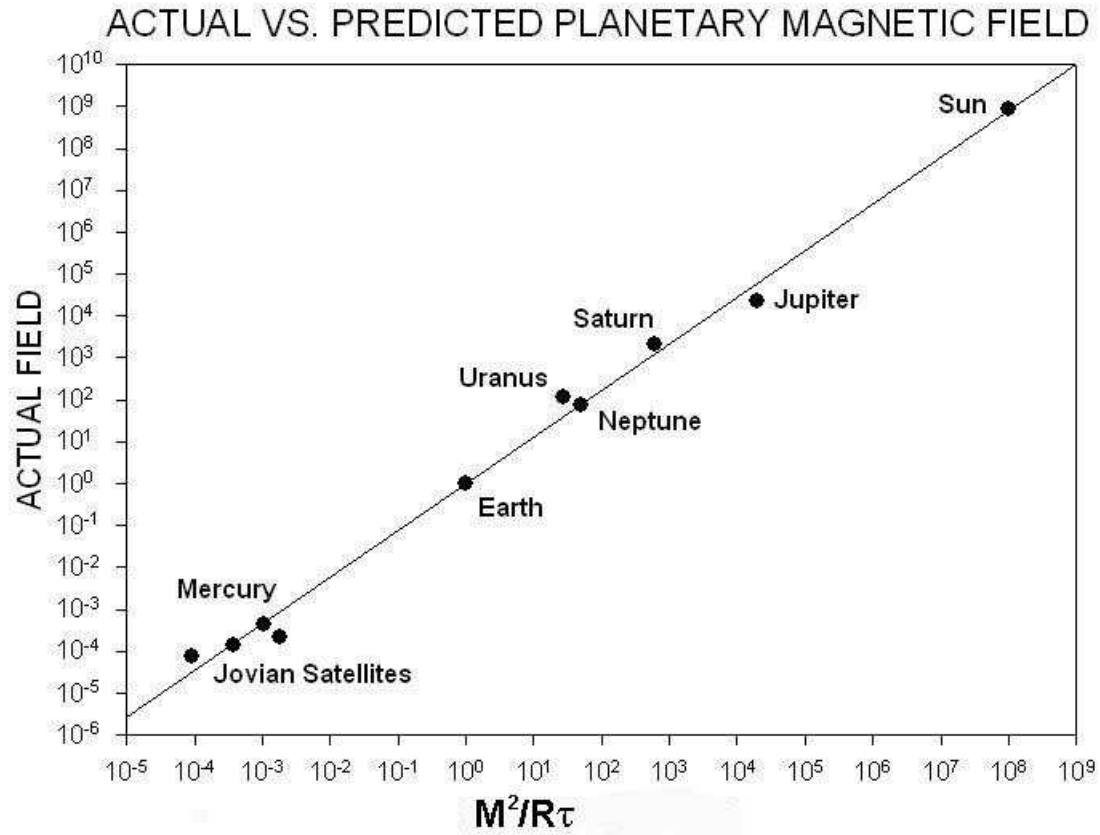


Fig. 3.— Graph for actual vs. predicted magnetic fields for the planets and satellites with detected global magnetic field. The actual and predicted field for the Earth is 1 to show that the scaling predicted by the model agrees with experimental scaling.

The process of magnetic field generation by a pressure gradient is not new. In 1955, Inglis (1955) consider this process and calculated the order of magnitude of the magnetic field. He stated that the generated field was eighteen orders of magnitude lower than the actual magnetic field. In the same direction, Merrill & McElhinny (1983) indicate that this conclusion was predictable since the pressure gradient effect would not exhibit inversions. But there are some objections about these conclusions, and we try to expose them in this section.

The first argument to be objected is that this effect could not exhibit polarity reversals. In the previous section we propose a mechanism with which the polarity could be achieved. The mechanism is the equilibrium between the loss of energy by Joule heating and the gain in energy due to the orientation of the outer shell respect to the inner shell.

The second argument needs a more extensive explanation. Inglis, in his considerations, used statistical mechanics to estimate the voltage difference between the external and the internal parts of the Earth's core. He obtained that the pressure in the inner core would raise the Fermi energy of the iron in  $2eV$  in relation to the outer part of the core. The effect in the electrons would be that these ones tend to accumulate in the outer core because their energy is lower. The change in the charge density creates an electric field that pulls the electrons back to the inner core until equilibrium is reached. At this point, Inglis considered that an electron needs a voltage difference of  $2V$  between the inner and outer part of the core. Although this would be correct in vacuum, he doesn't considered the fact that these electrons are in a material medium. As is well known from Sommerfeld theory of metals, the Fermi energy increase in a metallic wire subjected to an electric field is (Ashcroft & Mermin (1976); Kittel (1996)),

$$\delta k = \frac{q}{\hbar} E \tau \quad (27)$$

where  $k$  is the Fermi momentum,  $q$  the charge of the particle (electrons in our model),  $E$  the modulus of the applied electric field, and  $\tau$  the time between collisions of the particle in the medium. The time between collisions is related to the mean free path through the Fermi velocity  $\tau = \lambda/v_F$ , where  $v_F$  is the Fermi velocity. If we use these relations in the opposite direction, we obtain the value of the field generated by a shift in the Fermi energy,

$$E = \frac{2\Delta E_F}{q\lambda}. \quad (28)$$

The final result is that the applied field has to be high enough to give  $\Delta E_F$  of energy in a distance of the order of the mean free path and not on the complete core thickness.

The difference between both sizes is 14 to 16 orders of magnitude. Considering that in his discussion he obtained that the field created was 18 orders of magnitude lower than the actual one, we can view that this effect could give a considerable field. In the next section we refine these calculations and obtain the correct value for the magnetic field.

Stevenson (1974) also stated that the Ohmic dissipation would be too large to maintain this effect. In fact there are no currents in the process of charge separation and so there is not dissipation. The thermal velocity distribution is isotropic so the amount of electrons that go inward is the same as the amount of electrons pointing outward. The electrons going into the core are accelerated and when they impact with an ion they give the excess of energy to it by thermalization; in contrast, the electrons going outward, are deaccelerated and when they impact with an ion they have less energy than the medium and by the thermalization process they gain energy. The resulting effect is the absence of an energy release. The only process that generate heat dissipation is the one that makes the outer shell tilt respect to the inner shell, but in this process the velocities are small and the dissipation is low. In the numerical results section we try to make an approach to compute this value.

### 3. Numerical Results

Using a similar statistical mechanic approach like the one used by Inglis we compute the predicted magnetic field for the Earth. We start by calculating the mean energy for the electrons of the core under a pressure  $p$ . For the pressure we use the value obtained from hydrostatic equilibrium (1)

$$E_F = \frac{p}{n} = \frac{3GM^2}{8\pi R^4} \frac{1}{n}, \quad (29)$$

where  $n$  is the numerical free electron density of iron in the core. We use this relation in the Sommerfeld relation obtained in (28) and substitute the result in the relation (17) to obtain the amount of charge separated in the core.

$$Q = C\Delta RE = 4\pi\alpha\beta\epsilon \frac{3GM^2}{8\pi R^2} \frac{\Delta n}{n^2} \frac{1}{q\lambda}, \quad (30)$$

where  $q$  is the fundamental charge and  $\lambda$  the mean free path for electrons in the core. Instead of the value of the mean free path, we use the more usual value of the conductivity. With this objective we use the Drude-Sommerfeld relation between conductivity and mean free path, to write

$$\lambda = \frac{\sigma m_e v_F}{n q^2} = \frac{\sigma}{2n q^2} \sqrt{\frac{3Gm_e}{\pi n}} \frac{M}{R^2}, \quad (31)$$

where  $\sigma$  is the conductivity,  $v_F$  the Fermi velocity, and  $m_e$  the electron mass. By substitution of (30) and (31) on the magnetic dipole moment we obtain,

$$d = \frac{2\pi}{3} \frac{R^2}{\tau} Q = \left( q\epsilon \sqrt{\frac{4\pi^3 G}{3m_e}} \right) \alpha\beta \frac{\Delta n}{n} \frac{\sqrt{n}}{\sigma} \frac{MR^2}{\tau} \quad (32)$$

The values we use in the last relation for the conductivity and the permittivity are,  $\sigma = 10^5 \text{ S/m}$  and  $\epsilon = \epsilon_0 = 8,85 \cdot 10^{-12} \text{ F/m}$ .

For the value of the increment in the numerical density we assume that the difference in the numerical density of electrons in the core is proportional to the difference in the numerical density of iron atoms so is proportional to the difference in the mass density

$$\frac{\Delta n}{n} = \frac{\Delta \rho}{\rho} = \frac{(12,8 - 9,9) \text{ g/cm}^3}{12,8 \text{ g/cm}^3} = 0,22. \quad (33)$$

The free electron density at the center of the Earth core will be

$$n_i = \frac{\rho_i}{\rho_o} n_o = \frac{12,8 \text{ g/cm}^3}{7,85 \text{ g/cm}^3} \cdot 1,7 \cdot 10^{29} = 2,77 \cdot 10^{29} \text{ m}^{-3} \quad (34)$$

where  $n_i$  and  $n_o$  are respectively the free electron density in the core and at the surface of the Earth and  $\rho_i$  and  $\rho_o$  the corresponding mass densities. Using these values and the values for the mass, radius and period of rotation of the Earth we obtain the predicted value for the dipole moment of the Earth,

$$d = 3,03 \cdot 10^{22} \text{ A m}^2. \quad (35)$$

The actual value of the Earth dipole moment is  $7 \cdot 10^{22} \text{ A m}^2$ . We can see that the value predicted by the model is of the same order of magnitude as the actual value. This is in contradiction with the previous results that stated that this mechanism couldn't give a value close to the experiment.

A remarkable thing that must be noted is that the model obtains directly the wide known relation about magnetic dipole moment and the moment of inertia of the object  $d \propto I = MR^2/\tau$ .

Returning to the subject of ohmic dissipation we could make an approach to the value of the power dissipated by a shell moving respect to the Earth. The less favorable case would be the one in which the outer shell rotates in opposite sense of the Earth rotation. In this case the heat generated by ohmic dissipation would be

$$P_\Omega = \frac{\mathbf{j}}{\sigma} V = \frac{3Q}{(\beta R)^2 \tau}. \quad (36)$$

where  $\mathbf{j}$  is the current density. In the last relation we assumed a constant distribution of a charge  $Q$  over a sphere of radius  $\beta R$  spinning at an angular velocity of  $4\pi R/\tau$  respect to the Earth. Substituting 30 and 31 into 37 we obtain

$$P_{\Omega} = 144\pi^2\alpha^2\beta\frac{G}{m_e}\frac{\epsilon^2q^2\Delta n}{\sigma^3}\frac{M^2}{R\tau^2}. \quad (37)$$

In the case of the Earth, this value is 129 *MW*. This value is far lower than the values of actual heat dissipation of the Earth that are of the order of *TW*.

For the extension of this result to the rest of the planets we need the values of the conductivity and the free electron density for all of them. Even though there has been progresses in calculating the thermodynamical properties of the Earth core and mantle (Stevenson (1981); Vocadlo & Dobson (1999); Alfe et al. (2002, 2001); Shankland et al. (1993); Xu et al. (1993)), we have not yet the detailed values of the transport coefficients of the objects of the Solar System. In order to obtain such values for the Solar System, we must make some assumptions about the value of

$$\frac{\Delta n}{\sigma\sqrt{n}} \simeq \frac{\sqrt{n}}{\sigma} \simeq \sqrt{n}. \quad (38)$$

were we assumed that in the Drude-Sommerfeld model for metals, the conductivity is directly proportional to the numerical density. This value is not very different in different planets.

From these values we can calculate the ratio ( $d/d_e$ ) of the predicted magnetic dipole moment for the planet  $d$  with the predicted dipole moment for the Earth  $d_e$ , that is, how much stronger is the field predicted for the object than that predicted for the Earth.

In table 3 are listed the values for the radii, masses and revolution periods ( $R, M$  and  $\tau$ ) of different objects of the Solar System, relative to the values of the Earth ( $R_e, M_e$  and  $\tau_e$ ).

The values presented in table (3) show that the theory predicts significative intense fields to that objects that in fact have a significative measured field, while those objects that have not strong observed fields have a small predicted value. Moreover, the theory predicts a field strength for the Sun. The model also predicts global magnetic fields for objects like Venus and the Moon. If we calculate the magnetic field produced by these dipoles over the surface we obtain magnetic fields of 100 *nT* in the surface of Venus and 19 *nT* over the Moon (see table 3). This values are compatible with the experimental data from Pioneer Venus Orbiter and Apollo expeditions. In the case of Venus, the effect of the magnetic dipole moment could be in superposition with the magnetic effects coming from ionosphere since the observations of magnetic fields of several tens of *nT* made by the probe are at the same altitude as the

<b>Object</b>	$R/R_E$	$M/M_E$	$\tau/\tau_E$	$d/d_E$
Sun	110	$3,33 \cdot 10^5$	28	$10^7 - 10^8$
Mercury	0.383	0.0553	58.8	$3.85 \cdot 10^{-4}$
Venus	0.949	0.815	243.7	0?
Earth	1	1	1	1
Moon	0.273	0.0123	27.4	0?
Mars	0.533	0.107	1.03	0?
Jupiter	11.21	317.7	0.415	$2 \cdot 10^4$
Io	0.286	0.0150	1.77	$1.03 \cdot 10^{-3}$
Europa	0.245	0.0080	3.56	$8.97 \cdot 10^{-5}$
Ganymede	0.413	0.0248	7.17	$1.79 \cdot 10^{-3}$
Callisto	0.378	0.0180	16.73	?
Saturn	9.45	95.2	0.445	605
Titan	0.403	0.0225	15.99	?
Uranus	4.01	14.5	0.720	49.1
Neptune	3.88	17.1	0.673	27.7
Trito	0.212	0.0036	5.89	?
Pluto	0.187	0.0020	6.40	?
Caronte	0.0465	0.00027	6.40	?

Table 2: Table with the values of the radii, mass, rotation period and magnetic dipole moment of some celestial objects relative to the Earth.



<b>Object</b>	$d_p/d_{pE}$	$d_r/d_{rE}$	$A$
Sun	$3.60 \cdot 10^7$	$10^7 - 10^8$	$3.6 - 0.36$
Mercury	$1.35 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.34
Venus	$3.01 \cdot 10^{-3}$	0?	?
Earth	1	1	1
Moon	$1.98 \cdot 10^{-5}$	0?	?
Mars	$2.97 \cdot 10^{-2}$	0?	?
Jupiter	$2.42 \cdot 10^5$	$2 \cdot 10^4$	1.21
Io	$4.41 \cdot 10^{-4}$	$1.03 \cdot 10^{-3}$	0.43
Europa	$7.41 \cdot 10^{-5}$	$8.97 \cdot 10^{-5}$	0.83
Ganymede	$2.08 \cdot 10^{-4}$	$1.79 \cdot 10^{-3}$	0.12
Callisto	$5.13 \cdot 10^{-5}$	?	?
Saturn	2281	600	3.80
Titan	$7.86 \cdot 10^{-5}$	?	?
Uranus	113	50	2.26
Neptune	115	25	4.6
Trito	$1.03 \cdot 10^{-5}$	?	?
Pluto	$3.65 \cdot 10^{-6}$	?	?
Pluto	$2.47 \cdot 10^{-7}$	?	?

Table 3: Table that shows the predicted value of the magnetic dipole moment respect the one predicted for the Earth, the actually observed dipole, and the ratio of both columns, that shows the correlation between the predicted and the real magnetic fields. The values for the Earth are 1 to show the agreement of the actual scaling with the predicted one.

ionosphere. As a final point we can view that the model predicts a magnetic field for the great satellites of the Solar System like the galilean satellites of Jupiter. The fields predicted also agree with the observed magnetic field (Olson (1997)). Similar magnetic fields are obtained for other satellites like Titan. Cassini-Huygens mission to Saturn will measure the magnetic field of Saturn and Titan for which we lack experimental values from Voyager missions. We also see in figure 2 the great correlation between the predicted and the actual value of the magnetic field. In the plot (2), the value for the predicted and existing field for the Earth is set to 1 to show the agreement between the scaling in the model and the experimental one.

A final indication must be made about the multipolarity and the dynamic of the model. A spherical shell has higher order multipoles that are not computed here because it is out of the scope of an introductory paper, numerical calculations with the multipole distribution will be obtained shortly by the aid of computer models. Once multipolarity will be obtained, dynamics will be added by the introduction of magnetic interaction between both shells and with an outer field (planetary field in the case of a satellites or solar field in the case of a planet) and variability of pressure and temperature in the core of the object.

#### 4. Conclusions

The model presented in this paper describes, in a single theory, the magnetic field detected in several objects of the Solar System. The values obtained are close to the experimental ones. The model proposed could be used to break the initial symmetry of the system in the numerical geodynamo models. These models predict, also, a tilt of the magnetic dipole moment respect to the axis of rotation. This tilt could depend on the properties of the materials that fill the shell, like conductivity. Moreover, we can see that following this proposal, the planets with higher tilts of the magnetic dipole have higher quadrupole moments, because the total field would be generated by two crossed dipole magnets. This feature of the field agrees with the observed magnetic field of Uranus and Neptune (Connerney & Acua (1987, 1991)) and with the fact that the magnetic field of Saturn has a lower quadrupole moment (Acua et al. (1983)). The model can also explain the observation that the quadrupolar component in the Earth magnetic field becomes more important in the transition between normal and reversed polarity of the field (Schneider & Kent (1988)). As the field is generated by the effect of two concentric spherical shells, higher-order multipoles are predicted by the theory (a rotating sphere is not only dipolar or quadrupolar). The calculation of these higher orders is left because it is out of the scope of an introductory paper. The only planet unexplained by this model is Mars, but notice that the zero field could be an extreme case where the outer and the inner shell have opposite magnetic dipole

<b>Object</b>	$B_s$ (nT)
Sun	923000
Mercury	72
Venus	101
Earth	30000
Moon	30
Mars	4200
Jupiter	465000
Io	9100
Europa	2400
Ganymede	1400
Callisto	460
Saturn	76000
Titan	570
Uranus	34000
Neptune	57000
Trito	520
Pluto	17
Caronte	73

Table 4: Table that shows the predicted value of the magnetic field over the surface of the planet. This value is calculated from table (3) with ( $d_r = Ad_{rE}$ ) and using a value of  $d_{rE} = 7 \cdot 10^{22} Am^2$  for the Earth

values.

Magnetic fields for the galilean satellites are also predicted. Some theories say that the fields for these satellites are explained by the Jupiter magnetic field induction, and the reason for this hypothesis is that the fields of the satellites change of direction following the changes of Jupiter’s field. This reason is not enough to think that the field is induced. The field could be created as is explained in the model and, once created, it moves like a compass near a magnet.

The article also calculates the composition dependence of the model. The result is that the field depends of the inverse of the root of the numerical density. This dependence shows that the influence of the composition is not too high because increasing 10 times the numerical density only generates a multiplication of 0.3 of the magnetic field. Even though the composition of the nuclei of the several planets could be very different, its numerical densities couldn’t variate enough to change the order of magnitude of the generated fields.

The model could be generalized to other objects in the Universe like the main sequence stars and the neutron stars, objects where the gravitational effects are higher than in the planets. Even though these objects are surely not metallic in its core, charged particles could be found like electrons that could behave like the electrons in the model. In objects like neutron stars, geodynamo is surely less probable due to the ultra high density of the entire object.

In conclusion, the theory achieves with a thermodynamic model a scaling law that agrees with the values of the Solar System magnetic fields. Even though the accepted theory for the generation of the magnetic fields is the geodynamo, this thermodynamic model could be used to explain the order of magnitude and the scaling of the actual fields, fact that couldn’t be explained with the current theory.

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